Self-fulfilling Currency Attacks with Biased Signals

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Abstract

Morris and Shin (1998) have shown recently in a model of self-fulfilling currency crises, that when removing the assumption of common knowledge among speculators, multiplicity of equilibria disappears. In their model speculators are assumed to receive unbiased private signals about the true state of the fundamentals of the economy. We consider, instead, three different cases: a) signals are positively biased but speculators are not aware of the bias; b) the private sector believes that signals are positively biased, although they are not; c) in presence of a positive bias, speculators are aware of it. While case c) does not modify substantially Morris and Shin's result, cases a) and b) produce respectively a ‘credibility bonus’ and a ‘non-credibility malus’, that are strikingly similar to those obtained in the literature on credibility and on target zones. A whole range of equilibria becomes then possible, depending not only on the size of the standard error in the distribution of the signal, but also on the size and sign of the bias and, more interestingly, on the expectations of the private sector about the bias.

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1. Introduction

The literature on speculative attacks has been characterized by two different approaches. In the first, initiated by Krugman (1979) and by Flood and Garber (1984), speculative attacks result from the exogenous, divergent policy followed by the monetary authority. It is assumed that the private sector is able to verify whether money creation (which determines the exchange rate), is consistent with the central bank's commitment to fixed exchange rates. As soon as the state of the fundamentals weakens, the private sector reacts in order to avoid the losses resulting from an unexpected devaluation, and the central bank is forced to abandon the parity in advance with respect to the natural exhaustion of foreign reserves. There exists, then, a unique critical level for the fundamental, below which everybody attacks the currency and above which nobody does it.

In the second approach (Obstfeld, 1986, 1995, 1996), far from being the outcome of markets punishing undisciplined policymakers, speculative attacks may arise simply because of self-fulfilling expectations: given that the central bank's optimal policy is determined endogenously, speculators realize that if they attack the currency, the fixed exchange rate system will be abandoned, so that their prophecy becomes self-fulfilled.

In particular, Obstfeld (1996) proves the existence of three different regions for the ‘fundamental’ (denoted by $\theta$) of an economy: one of instability, where $\theta$ is in such a bad state that the monetary authority finds too costly to defend the parity and every speculator attacks to make profits or to avoid losses; one of stability, where $\theta$ is in very good shape so that no speculator has convenience to launch an attack on the currency and no devaluation takes place; and finally, the so called ‘gray area’, where anything can happen: the exchange rate will be maintained if only ‘few’ speculators attack, while the central bank will be obliged to devalue if a ‘high’ proportion of speculators attack. Multiplicity of equilibria, however, is due to the fact that everybody is assumed

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1 See Jeanne (2000) for a recent survey on the literature on speculative attacks.
to know that the state of the fundamentals is exactly included in the gray area.

Morris and Shin (1998) argue that when the assumption of common knowledge is removed and some asymmetric information between speculators and central bank is introduced, so that the central bank perceives correctly the state of the fundamentals while the private sector is uncertain about it and only receives unbiased private signals, the gray area vanishes and a unique equilibrium emerges in the model that they consider.\(^2\)

Their model however, hinges crucially on the assumption that private signals are unbiased. We remove such an assumption and consider the possibility that they may be biased. Our results contrast with the main conclusions drawn by Morris and Shin, and are in line with the evidence according to which speculative attacks seem to depend more on private sector's expectations than on the actual state of the fundamentals.

Our paper is structured as follows: in section 2 we present Morris and Shin's model; our extension is illustrated in section 3; in section 3.1 we introduce biased private signals; the different cases that we consider are listed and justified in section 3.2, while they are analyzed in detail in sections 3.3, 3.4 and 3.5; section 4 contains a discussion of our results and in section 5 we make some concluding remarks.

2. Morris and Shin's model

Morris and Shin (1998) assumes that the prior distribution of the fundamental is Uniform between 0 and 1, i.e. \(\theta \sim U[0, 1]\). Given a certain value \(\theta\) for the fundamental, the private signal, \(x\), received individually by each speculator, is also assumed to be uniformly distributed in the interval \([\theta - \varepsilon, \theta + \varepsilon]\), so that \(f(x|\theta) = 1/(2\varepsilon)\) and \(E(x|\theta) = \theta\), i.e. the signal received by speculators is an

\(^2\) Morris and Shin's model is at the origin of a growing body of literature, which has concentrated mainly on the distinction between private and public signal. Corsetti (1998), for example, stresses that common knowledge, and therefore the existence of multiple equilibria, would result from the presence of a precise public signal and no private signal, while the absence of common knowledge, and therefore a unique equilibrium, is obtained when no public information is available and only private signals exist. He argues however, that the assumption made by Morris and Shin (1998) that speculators receive different (private) signals (although identically distributed), does not exclude the presence of public information since each speculator might interpret it in a different way.
unbiased estimate of the "true" value of the fundamental. From the previous assumptions, it follows that once a speculator receives a given signal \( x \), she knows that the fundamental \( \theta \) will lie in the interval \([x - \varepsilon, x + \varepsilon]\), so that \( f(\theta|x) = 1/(2\varepsilon) \) and \( E(\theta|x) = x \).

They define with \( \pi(x) \) the proportion of speculators attacking the currency for any given signal \( x \), \((\pi'(x) < 0)\). Given the value of the fundamental, the expected proportion of speculators attacking the currency, \( s(\theta, \pi) \), is calculated by averaging \( \pi(x) \) over all possible values that signal \( x \) can take:

\[
s(\theta, \pi) = E[\pi(x) \mid \theta] = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x) f(x \mid \theta)dx \tag{2.1}
\]

The central bank will devalue the exchange rate if and only if \( s(\theta, \pi) \geq a(\theta) \), where \( a(\theta) \) is the proportion of speculators that, for any given level of the fundamental, is sufficient to oblige the monetary authority to abandon the peg of the currency.

For any speculator, the convenience to launch an attack depends on her pay-off, given by:

\[
h(\theta, \pi) = \begin{cases} e^* - e(\theta) - t & \text{if } s(\theta) \geq a(\theta) \\ -t & \text{if } s(\theta) < a(\theta) \end{cases} \tag{2.2}
\]

where \( e^* \) represents the level of the fixed exchange rate, \( t \) is the fixed cost of attacking, and \( e(\theta) \) stands for the ‘shadow’ value of the exchange rate prevailing when the exchange rate floats freely (the floating exchange rate refers to the number of units of foreign currency necessary to buy one unit of domestic currency).

However, since speculators estimate \( \theta \) through \( x \), they have to consider the expected pay-off, \( u(x, \pi) \), which is calculated by averaging \( h(\theta, \pi) \) over all possible values of the fundamental that may yield the observed value of the signal:\(^4\)

\[
u(x, \pi) = E[h(\theta, \pi) \mid x] = \int_{[0, e^*] \cap [x-\varepsilon, x+\varepsilon]} [e^* - f(\theta)] f(\theta \mid x) d\theta - t \tag{2.3}
\]

After proving the existence of a unique critical level \( x^* \) for the signal, strategy \( \pi \) becomes:

\(^3\) It is assumed that \( 2\varepsilon < \min[\theta, 1 - \theta] \)

\(^4\) Morris and Shin denote the region of integration of the \( h() \) function by \( A(\pi) \cap [x-\varepsilon, x+\varepsilon] \) where \( A(\pi) = \{ \theta : s(\theta) \geq a(\theta) \} \). Our notation is simply based on their result that \( A(\pi) = [0, \theta^*] \).
\[ \pi(x) = I_{x_\ast}(x), \quad \forall x, \text{ where} \]
\[ I_{x_\ast}(x) = \begin{cases} 1 & \text{if } x < x^\ast \\ 0 & \text{if } x \geq x^\ast \end{cases} \quad (2.4) \]

It follows that speculators will only be indifferent between attacking and not attacking when:
\[ u[x, I_{x_\ast}(x)] = 0 \quad (2.5) \]

Referring to the value taken by the fundamental, it turns out that when \( \theta < x^\ast - \varepsilon \), no speculator will receive a private signal higher than \( x^\ast \), so that everybody is expected to join the speculation, i.e. \( s[\theta, I_{x_\ast}(x)] = 1 \); the opposite case, in which nobody attacks, so that \( s[\theta, I_{x_\ast}(x)] = 0 \), is obtained when \( \theta \geq x^\ast + \varepsilon \). Finally, the gray area is given by \( x^\ast - \varepsilon \leq \theta < x^\ast + \varepsilon \), where the fundamental is such that some speculators might receive a signal \( x < x^\ast \) and some others a signal \( x \geq x^\ast \). The expected proportion of speculators launching an attack in this region is given by:
\[ s[\theta, I_{x_\ast}(x)] = \int_{\theta - \varepsilon}^{x^\ast} f(x \mid \theta)dx + \int_{x^\ast}^{\theta + \varepsilon} f(x \mid \theta)dx = \frac{1}{2} - \frac{1}{2\varepsilon} (\theta - x^\ast), \quad (2.6) \]

so that the central bank will be indifferent between devaluing and not devaluing the currency when:
\[ s[\theta, I_{x_\ast}(x)] = a(\theta) \quad (2.7) \]

Equations (2.5) and (2.7) taken together allow to calculate \( x^\ast \) and \( \theta^\ast \) (see figure 1).

### 3. Morris and Shin's model with biased private signals

#### 3.1. Introducing biased private signals

Like in Morris and Shin (1998), we assume that the prior distribution of the fundamental is Uniform between 0 and 1 i.e. \( \theta \sim U[0, 1] \). As for the private signal \( x \) received by speculators, however, the apparently innocuous assumption of unbiasedness is far from being justified. We think that it might be of some interest to verify how the results change when considering the possibility that private signals may be biased and/or believed to be biased by the private sector\(^5\).

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\(^5\) We concentrate only on the existence of a positive bias; of course, a negative one could also be considered.
A more general approach should allow, then, for the possibility that for a given state of fundamentals $\theta$, the biased private signal received by agents, that we denote with $x'$, may lie in the interval $[\theta + \gamma - \varepsilon, \theta + \gamma + \varepsilon]$, where the bias $\gamma$ may be either positive or negative\(^6\). When $\gamma > 0$, $E(x'|\theta) = \theta + \gamma > \theta$, while $f(x'|\theta)$ is still $1/(2\varepsilon)$ in the considered interval (see figure 2). Moreover, in presence of a biased private signal, it turns out that the posterior distribution of $\theta$ is Uniform in $[x' - \gamma - \varepsilon, x' - \gamma + \varepsilon] = [x - \varepsilon, x + \varepsilon]$, $\forall \gamma$, so that we still have that $E(\theta|x') = x$, and $f(\theta|x') = 1/(2\varepsilon)$.

However, if the bias is unperceived, so that it is not appropriately discounted, the posterior distribution of the perceived fundamental, $\theta'$, is uniform in $[x' - \varepsilon, x' + \varepsilon]$, so that $E(\theta'|x') = x'$, i.e.

\(^6\) Consistently with Morris and Shin's model (1998), we keep assuming that the signal is uniformly distributed. However, we stress that this is an excessive simplification of reality. It would be more realistic to assume that the signal is normally distributed, as in Morris and Shin (1999) and we plan to consider this hypothesis in future works.
the fundamental is over-estimated by $\gamma$, and $f(\theta'|x') = 1/(2\varepsilon)$ (see figure 3).\footnote{\textsuperscript{7}}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Conditional density function of a positively biased signal}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Posterior density function of the fundamental perceived by speculators when signal is positively biased and speculators are not aware of the bias.}
\end{figure}

\subsection*{3.2. Actual and expected bias: three different cases}

Our assumption of positively biased private signals within the framework of Morris and Shin's model, leads us to distinguish among three different situations depending on the presence or the absence of the bias ($\gamma > 0$ and $\gamma = 0$, respectively), and on the expectation or non-expectation of the bias by the private sector ($\gamma^e > 0$ and $\gamma^e = 0$, respectively), where $\gamma^e$ indicates the bias expected by the private sector. In particular, we consider the three following situations, that we compare with the case analyzed by Morris and Shin (henceforth, the reference case):

a) The bias is present but the private sector does not recognize it ($\gamma > 0$, $\gamma^e = 0$);

b) The bias is not present, but the private sector believes that the received signal is biased ($\gamma = 0$, $\gamma^e > 0$);

\footnote{In figures 2 and 3 $\varepsilon$ is greater than $\gamma$. Our reasoning, however, applies also when $\gamma \leq \varepsilon$, since we do not make any...}
\( \gamma^e > 0 \);

c) The bias is present and the private sector is aware of it (\( \gamma = \gamma^e > 0 \)).

In the reference case the bias is not present and the private sector does not expect it (\( \gamma = \gamma^e = 0 \)). In order to study the cases listed above, it will be convenient to parameterize the \( s(\ ) \) function as follows:

\[
\begin{align*}
  s[\theta, I_k(x), \gamma - \gamma^e] &= \begin{cases} 
    s[\theta, I_k(x), \gamma] & \text{in case a)} \\
    s[\theta, I_k(x), \gamma^e] & \text{in case b)} \\
    s[\theta, I_k(x), 0] & \text{in case c) and in the reference case}
  \end{cases}
\end{align*}
\]

where \( k \) indicates the critical value of the signal. As we shall prove, such a value might be different from \( x^* \), depending on the case we consider.

Cases a) and b), however, might seem to be problematic. As a matter of fact, the central bank might well be tempted to release positively biased public signals, as it is often observed in reality. If, in turn, producing biased signals is the dominant strategy for the central bank, when expectations are taken rationally the private sector should expect the bias, so that case a) (and, for symmetric reasoning, case b)), should be ruled out. The conclusion might be then that, in line with the standard result obtained in the time inconsistency literature, the unique sub-optimal Nash equilibrium admitted by this simple game is given by the couple (\( \gamma > 0, \gamma^e > 0 \)) corresponding to our case c) which, as we will see, leaves unchanged the critical level \( \theta^* \) obtained in Morris and Shin's reference case.

Two objections however can be made to this conclusion. The first is that even when the private sector recognizes the incentive for the central bank to provide biased signals, it might well be that the expected bias does not reflect exactly the actual size of the bias. In other words, the private sector might suffer of bounded rationality. In our view, both the recent accounting scandals in the USA, and the bubble affecting the world stock markets during the second half of the 1990s, provide assumption concerning the relationship between these parameters.
two clear examples of the possible difference between $\gamma$ and $\gamma^e$.  

The second and more important objection is that the case analyzed by Morris and Shin can be interpreted as one in which no public information is available and only private signals exist. In such a situation, no role is left to the public signal provided by the central bank, so that the issue of time inconsistency cannot arise. Private sector’s expectations, then, might simply reflect exogenous shifts in the so-called market sentiment. Several interpretations of the Asian crisis, for example, stress the role of ‘contagion’ and ‘wake up calls’, so that fundamentals were generally believed as perfectly sound as long as no country in the region was hit by a crisis. As soon as this happened, however, the crisis spread to the neighboring countries simply because the private sector suddenly assigned a positive probability to the fact that some pieces of information might have been missing.

3.3. A positively biased signal, believed to be unbiased by the private sector

In this section we consider the case in which $\gamma > 0$ but $\gamma^e = 0$. Given the assumption that speculators do not realize the bias, we expect its presence to produce some real effects on the critical level of the fundamental.

**Proposition 1.** By comparing case a) ($\gamma > 0$, $\gamma^e = 0$) with the reference case, it turns out that the critical level of the unbiased signal is still unique and is reduced by $\gamma$, while the critical level of the fundamental is reduced by less than $\gamma$.

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8 Our analysis might have been conducted by distinguishing among three more general different cases ($\gamma - \gamma^e = 0$, $\gamma - \gamma^e > 0$ and $\gamma - \gamma^e < 0$), with no restrictions on the value that both $\gamma$ and $\gamma^e$ can take. Modelling in such a way would have made even clearer the difficulty for the private sector to anticipate exactly the optimal bias produced by the central bank, so as to point out that cases a) and b) are far more likely to occur than case c). However, while the notation would have become more cumbersome, the essence of our reasoning would not have changed.

9 See note 2.

10 One of the main puzzles in economics is to understand why markets’ behavior changes at some point; some authors reject the idea that there may be something which is left unexplained (what Keynes called ‘animal spirits’), because it conflicts with their “theoretical scruples against indeterminacy” (Morris and Shin, 1999). As it should be clear we believe instead that the role played by expectations should not be ignored, even when they cannot be endogenized.

11 These observations lead us to a more general critique against the assumptions made in Morris and Shin's model (and in most of the literature on speculative attacks), namely that the state of the fundamentals allows to identify a unique
Proof.

In order to prove the uniqueness of the critical level of the signal, it is enough to replace, in the steps followed by Morris and Shin (1998), variable \( x \), which is unbiased, with variable \( x' \), which is biased: \( x' = x + \gamma \). Lemma 1, Lemma 2 and Lemma 3 in Morris and Shin follow directly. In particular, since the private sector is not aware of the bias, equations (8) and (9) in Morris and Shin (1998), become respectively:

\[
\begin{align*}
\inf\{x : \pi(x) < 1\} &\leq x' + \gamma \leq \sup\{x : \pi(x) > 0\} & (\text{where } x = \inf\{x : \pi(x) < 1\} \text{ and } x = \sup\{x : \pi(x) > 0\} \text{, see Morris and Shin, 1998, from which the definition of } x' \text{ and } x \text{ follows consequently}).
\end{align*}
\]

Since the arguments used by Morris and Shin in order to get from equation (6) to equation (9) apply to the biased signal \( x' \) (as it can be verified easily), it turns out that \( x + \gamma = x' = x + \gamma \), i.e. the critical level of the biased signal is unchanged with respect to the original Morris and Shin's case. This implies that the unique critical level of the unbiased signal is given by \( x = x^* - \gamma = \bar{x} \), i.e. it shifts to the left by \( \gamma \).

The intuition for this result is immediate; let us recall first that speculator's decisions to attack are strategic complements (see Lemma 1 in Morris and Shin, 1998). Since in presence of a biased signal a lower proportion of speculators is expected to attack conditional on \( \theta \), (as we will see below), the expected pay-off to attacking the currency conditional on the biased signal \( x' \) is lower than when the signal is unbiased. In other words, in presence of a biased signal, while the critical level of the signal in terms of \( x' \) remains unchanged to \( x^* \), the critical level in terms of \( x \) is equal to \( x^* - \gamma \), i.e. it is lower than in Morris and Shin's case since the \( u(.) \) function shifts to the left by \( \gamma \), as shown in figure 4. This completes the proof of the first part of proposition 1.\(^{12}\)

As for the second part of proposition 1, let us consider that when \( \theta < x^* - \gamma - \epsilon \), no speculator

\(^{1}\)shadow' exchange rate. This assumption does not seem realistic to us and this is precisely the reason why the private sector might interpret differently, at different points in time, situations that are apparently similar.

\(^{12}\) The same conclusion would be obtained by considering that in case a) equation (2.4) becomes:

\[
I_{x'}(x^*) = \begin{cases} 
1 & \text{if } x' < x^* \text{ i.e., } \left\{ \begin{array}{l} 
1 & \text{if } x < x^* - \gamma \\
0 & \text{if } x \geq x^* - \gamma 
\end{array} \right. 
\end{cases}
\]
will receive a (biased) signal higher than $x^*$, so that everybody is expected to join the speculation, i.e. $s[\theta, I_x^*(x'), \gamma] = 1$; the opposite case, in which nobody attacks, is obtained when $\theta \geq x^*-\gamma + \varepsilon$: in this case no speculator will receive a (biased) signal lower than $x^*$, so that $s[\theta, I_x^*(x'), \gamma] = 0$.

Finally, the gray area is given by $x^*-\gamma - \varepsilon \leq \theta < x^*-\gamma + \varepsilon$, where the fundamental is such that some speculators might receive a biased signal $x' < x^*$ and some others a biased signal $x' \geq x^*$.

Since for any value of the fundamental, the biased signals received by speculators lie in the interval $[\theta + \gamma - \varepsilon, \theta + \gamma + \varepsilon]$, the expected proportion of speculators launching an attack in this region is given by:

$$s[\theta, I_{x^*}(x'), \gamma] = \frac{x^*}{\theta + \gamma - \varepsilon} \int_{\theta + \gamma - \varepsilon}^{\theta + \gamma + \varepsilon} f(x'|x^*)dx' + \frac{0}{x^*} = \frac{1}{2} \left( 1 - \frac{1}{2\varepsilon} (\theta - x^* + \gamma) \right).$$
given that for any biased signal \(x' < x^*\), \(\pi(x') = I,\) \(x' = 1\) and for any \(x' \geq x^*\), \(\pi(x') = I,\) \(x' = 0.\)

To summarize:

\[
s[\theta, I_{x^*}(x'), \gamma] = \begin{cases} 
1 & \text{if } \theta < x^* - \gamma - \varepsilon \\
\int f(x' | \theta) dx' = \frac{1}{2} - \frac{1}{2e} (\theta + \gamma - x^*) & \text{if } x^* - \gamma - \varepsilon \leq \theta < x^* - \gamma + \varepsilon \\
0 & \text{if } \theta \geq x^* - \gamma + \varepsilon
\end{cases}
\]

(3.3)

This result allows us to conclude that the curve representing the expected proportion of speculators attacking the currency shifts to the left by \(\gamma\) with respect to the reference case. Therefore, the critical value of the fundamental, \(\theta^*\), which is obtained as a solution of the equation

\[
\frac{1}{2} - \frac{1}{2e} (\theta - x^* + \gamma) = a(\theta),
\]

is lower than the level \(\theta^*\), obtained in the reference case (see figure 5).

![Figure 5. The critical level of the fundamental when the private signal is positively biased, but the private sector is not aware of it (case a).](image)

The intuition for this result is immediate: when the private sector does not recognize the bias in the received signals, she over-estimates \(\theta\), so that the expected proportion of speculators attacking the currency, conditional on any given state of the fundamental \(\theta\), is systematically lower with

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\[\text{It is obvious that when considering the unbiased signal } x \text{ the above equation becomes:}\]

\[
s[\theta, I_{x, \gamma}(x)] = \int_{x^* - \gamma}^{\theta + \varepsilon} f(x | \theta) dx + \int_{\theta - \varepsilon}^{x^* - \gamma} f(x | \theta) dx = \frac{1}{2} - \frac{1}{2e} (\theta - x^* + \gamma),
\]

and gives the same result, since for any unbiased signal \(x < x^* - \gamma\), \(\pi(x) = I,\) \(x = 1\) and for any \(x \geq x^* - \gamma\), \(\pi(x) = I,\) \(x = 0.\)
respect to the case of an unbiased signal. As a result, the \( s(.) \) function shifts to the left and the non-attack region gets extended, i.e. even weaker levels of the fundamental become attacks-free. It should be noted however, that although the \( s(.) \) function shifts to the left by \( \gamma \), the critical level of the fundamental decreases by less than \( \gamma \), because of the positive slope of the \( a(.) \) function. The more \( a(.) \) depends on \( \theta \), the lower is the difference between \( \theta^* \) and \( \theta^*'. \) This completes the proof of proposition 1.

3.4. An unbiased signal, believed to be positively biased by the private sector

When \( \gamma = 0 \) but \( \gamma^e > 0 \), the private sector believes in the existence of a bias, while in reality signals are unbiased. As it will be shown, this is the worst possible situation for the monetary authorities.

**Proposition 2.** By comparing case b) \((\gamma = 0, \gamma^e > 0)\) with the reference case, it turns out that the critical level of the signal is still unique and is increased by \( \gamma \), while the critical level of the fundamental is increased by less than \( \gamma^e \).

**Proof.** The proof follows closely the one of section 3.3. In this case, however, we need to consider, in the steps followed by Morris and Shin (1998), variable \( x'' = x - \gamma^e \), where \( x \) is the unbiased signal perceived by speculators and \( \gamma^e \) is the bias that they expect. As a result, speculators will refer to the adjusted signal \( x'' \). Analogously to the previous case, the critical level of \( x'' \) is \( x^* \), whereas the critical level of the unbiased signal \( x \) is \( x^* + \gamma^e \). The proof of uniqueness of the critical signal is also analogous to the previous case. Therefore, it follows that: \( x = x^* + \gamma^e = x'' \), meaning that in presence of an expected bias \( \gamma^e \), when referring to the unbiased signal \( x \) the critical level shifts to the right by \( \gamma^e \) with respect to the reference case.

As for the second part of proposition 2, when \( \theta < x^* + \gamma^e - \varepsilon \), no speculator will receive a signal \( x \) higher than \( x^* + \gamma^e \), so that everybody is expected to join the speculation, i.e. \( s[\theta, I_{x^* + \gamma^e}(x), \gamma^e] = \)
1. The opposite case, in which nobody attacks, is obtained when \( \theta \geq x^* + \gamma^e + \varepsilon \); in this case no speculator will receive a signal \( x \) lower than \( x^* + \gamma^e \), so that nobody will be expected to attack, i.e. \( s[\theta, I_{x^*+\gamma^e}(x), \gamma^e] = 0 \). Finally, the gray area is given by \( x^* + \gamma^e - \varepsilon \leq \theta < x^* + \gamma^e + \varepsilon \), where the fundamental is such that some speculators might receive a signal \( x < x^* + \gamma^e \) and some others a signal \( x \geq x^* + \gamma^e \). The expected proportion of speculators launching an attack in this region is given by:

\[
s[\theta, I_{x^*+\gamma^e}(x), \gamma^e] = \int_\theta^{x^*+\gamma^e} f(x | \theta)dx + \int_{x^*+\gamma^e}^{\theta+\varepsilon} 0 \cdot f(x | \theta)dx = \frac{1}{2} - \frac{1}{2\varepsilon} (\theta - x^* - \gamma^e),
\]

given that for any unbiased signal \( x < x^* + \gamma^e \), \( \pi(x) = I_{x^*+\gamma^e}(x) = 1 \) and for any \( x \geq x^* + \gamma^e \), \( \pi(x) = I_{x^*+\gamma^e}(x) = 0 \).

To summarize:

\[
s[\theta, I_{x^*+\gamma^e}(x), \gamma^e] = \begin{cases} 
1 & \text{if } \theta < x^* + \gamma^e - \varepsilon \\
\int_\theta^{x^*+\gamma^e} f(x | \theta)dx = \frac{1}{2} - \frac{1}{2\varepsilon} (\theta - x^* - \gamma^e) & \text{if } x^* + \gamma^e - \varepsilon \leq \theta < x^* + \gamma^e + \varepsilon \\
0 & \text{if } \theta \geq x^* + \gamma^e + \varepsilon
\end{cases}
\]  

(3.5)

This result allows us to conclude that the curve representing the expected proportion of speculators attacking the currency shifts to the right by \( \gamma^e \). Therefore, the critical value of the fundamental, \( \theta^{*''} \), which is obtained as a solution of the following equation:

\[
\frac{1}{2} - \frac{1}{2\varepsilon} (\theta - x^* - \gamma^e) = a(\theta),
\]

is higher than the level \( \theta^* \) obtained in the reference case (see figure 6).

By following the observations made in case a), we conclude that \( \theta^* \) increases by less than \( \gamma^e \), due to the positive slope of \( a(\theta) \). This completes the proof of proposition 2.
3.5. A positively biased signal, fully recognized by the private sector

In this section we examine the case in which $\gamma^e = \gamma > 0$ and we shall prove the following proposition:

**Proposition 3.** By comparing case c) ($\gamma^e = \gamma > 0$) with the reference case ($\gamma^e = \gamma = 0$), it turns out that the unique critical level of both the signal and the fundamental are unchanged.

**Proof.** In this case the proof is trivial. Speculators receive a biased signal $x' = x + \gamma$, whose bias is correctly recognized, so that they adjust for it and refer to variable $x = x' - \gamma$. Since variable $x$ is unbiased, its critical level is simply $x^*$, as in the reference case. As a result, the $s(\cdot)$ function does not move so that the critical value for the fundamental is also unchanged.

This result allows us to unify case c) and the reference case, so that proposition 3 apply when $\gamma = \gamma^e \geq 0$.

Figure 7 summarizes the three different cases.

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14 Alternatively, if we refer to the biased signal $x'$, speculators will stop attacking at $x^* + \gamma$, rather than at $x^*$, so that:

$$s[\theta, I_{x^*} (x')] = \frac{\theta + \gamma + \varepsilon}{\theta + \gamma - \varepsilon}$$

$$s[\theta, I_{x^*} (x')] = \frac{1}{2\varepsilon} \int_{x^*}^{x^* + \gamma} f(x') dx' = \frac{1}{2\varepsilon} \left[ \frac{x^* + \gamma}{\theta + \gamma - \varepsilon} \right]$$

15 In cases a) and b), we might also wonder whether the couple $(x^*, \theta^*)$ is derived consistently by both speculators and central bank. It is certainly true, for example, that if the private sector is unaware of the positive bias in the signal, as in
4. Biased signals, target zones and credibility models

In case c) no difference with Morris and Shin's result is found. The existence of a positive bias is correctly perceived by the private sector, so that the critical level of the fundamental separating the instability from the stability region is unchanged.

In case a), given that the presence of the positive bias is not perceived by the private sector, the central bank obtains a ‘credibility bonus’. Since $\theta$ is over-estimated, its critical level decreases, so that the central bank moves into the stability area even with weaker fundamentals, compared to case c) and to the reference case analyzed by Morris and Shin. Such a gain is similar to the one emerging in the literature on target zones (Krugman, 1991), when the private sector fully believes in the commitment by the central bank to intervene in defense of the lower margin ($e$) of a given exchange rate band. In this case, when a negative random shock hits the fundamental, so as to move it back from its initial level $\theta_0$, (see figure 8), the currency $e(\theta)$, (which is also assumed to depend case a), it will stop attacking the currency even if the fundamentals are relatively weak. This situation might induce to conclude that speculator's strategy is not the best response to central bank's strategy, i.e. that the equilibrium would not be sub-game perfect, since speculators do not attack the currency when the fundamental is such that the central bank would devalue. The private sector, however, simply ignores this. Lack of sub-game perfectness would only arise if, being aware of the presence of the bias, the private sector did not attack for relatively low values of the fundamental. Speculators' strategy, then, is certainly the best response to central bank's strategy, given their available information and/or beliefs. In turn, central bank's strategy is the best response to speculator's strategy, since the central bank will defend the currency at $\theta^*$, the critical level of the fundamental for speculators. Similar reasoning applies to the case of...
positively on the state of the fundamental), rather than following the linear path prevailing when exchange rates float freely and leading downwards to the critical level of the fundamental $\theta^*$, approaches smoothly the lower edge of the band. As a matter of fact, the closer the exchange rate gets to its lower limit, the higher the probability of a central bank intervention to keep it inside the band: the expectation of a central bank intervention implies that the expected value of the exchange rate will be above the lower limit of the band. The policymaker enjoys then a ‘honeymoon’ effect, meaning that the region of stability widens, and the exchange rate remains inside the band when $\theta < \theta^*$, i.e. even for values of the fundamental that are weaker than $\theta^*$ (see Krugman, 1991 for further details).

Literature on target zones, however, comes to the conclusion that the opposite result may be obtained: the ‘honeymoon’ may suddenly end to be replaced by a ‘divorce’ (Bertola and Caballero, 1992). This means that when an exchange rate realignment, rather than intervention, is expected, the ‘smooth pasting’ result obtained in the original Krugman's paper turns into an explosive downward trajectory leading to the anticipated abandonment of the fluctuation band, so that the region of instability gets wider (for $0 < \theta < \theta^{*''}$ in figure 8). As a matter of fact, in this case the more the negative shock reduces the fundamental, the more likely, when approaching the lower limit of the target zone, is a downward realignment of the exchange rate (see Bertola and Caballero, 1992 for further details). ‘Honeymoon’ or ‘divorce’, then, like in our cases a) and b) respectively, depend on the state of expectations of the private sector: the same negative shock may cause opposite outcomes depending on whether the private sector expects either a central bank intervention or an exchange rate realignment.

an expected, although absent, bias in the received signals.
Figure 8: ‘Honeymoon’ effect ($\theta^*$) and ‘divorce’ effect ($\theta^{**}$) in a target zone model.

In the literature on credibility it is also possible to find examples in which there exist differences between expected and actual values: even when appointing a conservative policymaker or when increasing the degree of independence of the central bank, private sector's expectations may not adjust accordingly. After all, different governments get elected and different central bankers get appointed over time, so that the private sector might not recognize fully their true nature and incentives. In such a situation disinflation may turn into a long and painful process during which unemployment increases and inflationary expectations are hard to curb down, as it might have been the case for Europe during the Eighties (see De Grauwe, 1997 for further details). We obtain the same result in case b). In such a situation, the central bank's defense of the fixed exchange rate is more costly, being affected by a sort of ‘non-credibility malus’, exactly the opposite of the ‘credibility bonus’ that was gained in case a). As a matter of fact, in such a case only a higher level of the fundamentals will convince speculators that the region of instability has been abandoned.

5. Concluding remarks

In this paper we have presented a modified version of Morris and Shin's model, allowing for the presence of actual and/or expected positively biased signals. We have distinguished among three different cases in which the bias is present or is not present and speculators realize it or not.
The best result for the central bank is obtained when the private sector believes that the signals are unbiased while they are biased, so that the stability area gets wider. Such a ‘credibility bonus’ is also obtained in the literature on target zones, where the policymaker enjoys a ‘honeymoon’ effect.

The worst outcome for the central bank results instead when the private sector believes that the received signals are positively biased, while this is not the case: the ‘credibility bonus’ becomes a ‘non-credibility malus’. Once again, this result resembles closely those obtained in target zones models and also in the theory of credibility. In the first case, when the promise to intervene at the edges of a currency band is not believed and realignment is expected instead, the ‘honeymoon’ may turn into ‘divorce’. In the second case, if expectations do not adjust to the more restrictive policies followed by the policymaker, inflation is reduced slowly and only at a very high cost in terms of unemployment.

While it can be argued that the private sector might be able to recognize - and therefore neutralize - the incentives of the policymakers to provide positively biased signals (although a difference between actual and expected bias is still possible and likely), we have pointed out that Morris and Shin's result relies on private rather than public information, so that private sector's expectations may change independently from policymakers' incentives and ability to provide biased public information.

Our extension shows that, even when the assumption of common knowledge is removed, a whole range of possible equilibria may arise, depending not only on the size of the standard deviation of the signal, but also on the size and sign of its bias and above all on the particular state of private sector's expectations, which may still be highly unpredictable.

In conclusion, we believe that the sources of uncertainty represented by the actual and/or the expected bias (and also by the standard error of the distribution, on which we did not focus here), should be considered. When doing this, information supplied by signals becomes less accurate and speculative attacks will depend less on the state of the fundamentals and more on private sector's expectations.
References


P. Krugman (1999), Balance Sheets, the Transfer Problem and Financial Crises, Mimeo.


